THE ROLE OF WEALTH PREFERENCES IN THE EQUITY PREMIUM PUZZLE

Pat McPherron[1]

The relatively high (80%) US average consumption-income ratio is a critical element in resolving the combination of high average rates of real equity returns (7%) with relatively low real per capita consumption growth rates (2%) evident in the equity premium puzzle. The model of wealth preferences allows for calibrating the high consumption-income ratios in the US, showing that asset returns of 7% (10%) are necessary to generate average per capita consumption growth rates of 2% (3%). This model represents an important step in endogenizing the high ratios, whereas Mehra-Prescott optimality conditions indicates ratios less than 20% in the model economies.

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[1] *Address for correspondence*: Department of Economics, University of Connecticut, 341 Mansfield Road, Unit 1063, Storrs, CT 06269-1063. *Email*: pmcpherron@gmail.com. *Phone*: 407-340-5323.

1. Introduction

An equity premium puzzle results when Mehra and Prescott (1985) extend a constant relative risk aversion (CRRA) Lucas (1978) asset pricing model from an equilibrium consumption level to an equilibrium per capita consumption growth rate, supposedly endogenizing the required rate of return in the well-known dividend discount model. They calibrate consumption from US averages in order to observe asset pricing dynamics, but find their model generates virtually no risk premium for equity if assuming CRRA values in the interval [1, 2]. The normative assumption in the asset pricing model of an endowment economy is that the differential of utility in the current period will equal the expected differential of utility in the next period, such that the ratio of the total return over investment is a factor of next period's marginal consumption. The equity premium puzzle is the failure of this consumption-based capital asset pricing model with log utility preferences to generate the triplet of a smooth, and relatively low, average annual per capita consumption growth rate, an even lower average annual risk-free return, and the relatively high average annual rate of return on equity that is observable from 1889-1978.[2] Subsequently, hundreds of papers have generally confirmed the Mehra-Prescott results for different time periods, different time frequencies, alternative consumption bundles and asset classes.[3]

I assume a CCRA value of one (log utility) and identify the role of US average consumption-income ratios in restricting per capita consumption growth rates to levels consistent with observation, even when funded by real rates of return that are equal to or greater than US averages for risky assets from 1889-1978. A corollary of the model I develop is that assuming full consumption of income (dividend), as Mehra and Prescott do, indicates virtually no return for any asset class, otherwise the agent would invest a percentage of income. This corollary is consistent with their results, as they find a negligible premium for equity over an average real risk-free rate that is only one-half of one percent. In other words, any asset class they model should exhibit an average rate of return of nearly zero for log utility consumption preferences, if consuming all of income.

The observable consumption-income ratios (typically 80% or higher) in the national income and product accounts (NIPA) from 1929 to 2007 indicate that relatively low savings rates are a factor in

[2] A constant relative risk aversion value of one is the theoretical ideal of Arrow (1971); Chetty (2006), moreover, uses elasticities from labor economics to support values near one as optimal.
[3] Grossman and Shiller (1981) and Hansen and Singleton (1983) note high levels of risk aversion, but attribute it to peculiarities in the data set or specific model restrictions.

precluding the per capita consumption growth rates implied in the Mehra-Prescott optimality conditions.[4] In other words, the observable savings rate is insufficient to support the higher per capita consumption growth rates one expects from the Mehra-Prescott model at US averages for equity rates of return.[5] Conversely, the US average savings rate requires relatively high rates of return on assets to support even relatively low consumption growth rates. Relatively high asset rates of return and low consumption growth rates are a puzzle if one estimates the equity Euler equation in the Mehra-Prescott infinite horizon model and may be seen as the core of the "empirical failure" of a CRRA Lucas asset pricing model.

By contrast, I derive the consumption-income ratios for the Mehra-Prescott optimality conditions and find a relatively high rate of savings (in excess of 80% of income for asset rates of return at 6% or higher) is necessary to sustain the implied higher consumption growth rates over an infinite horizon.[6] This last result indicates that managers of corporations generally, at least to the extent that the S&P 500 returns correlate with typical behavior, tend to treat equity as an infinitely-lived asset in that the dividend-payout ratio is well within acceptable range implied by the Mehra-Prescott optimality conditions.[7] However, consumers do not live forever and typically do not save during their lifetimes at the rates necessary to emulate corporate dividend payout ratios.

I model only one asset class, which is risk-free, but assume rates of return comparable with US averages for risky assets from 1889-1978. Using a risk-free asset increases the optimal consumption growth rate, as does the assumption of zero net savings. I choose to model an economy in with these assumptions order to show that even *upper bounds of the rates of consumption growth* that can be expected in an economy with risky rates of return are still very low and consistent with observation.[8] In other words, I am allowing the growth rate of consumption to be higher than can be expected in a risky economy and yet the model generates consumption growth rates consistent with observation. Specifically, I calibrate the *wealth preferences* model with the high consumption-income ratios and derive the optimal consumption growth

[4] Equivalently, for wealth preserving agents, consumption-wealth can be replaced with consumption-income ratios. Additionally, note that wealth preferences are not wealth effects, as only the original endowment remains.

[5] For the remainder of the paper, consumption growth rate(s) refers to per capita consumption growth rate(s)

[6] NIPA data indicates the average savings to income ratio is 20% or less during the last 70+ years.

[7] Additional factors that impact on dividend payout ratios are outside the scope of the analysis here.

[8] The model also retains the restriction from Mehra-Prescott that the elasticity of intertemporal substitution is the inverse of the CRRA value.

rate, assuming a finitely-lived agent makes different consumption-savings decisions than an infinitely-lived agent.

The fact that agents know they will not live forever creates a terminal period consumption choice, but that particular time period is stochastic and there is also an issue of bequeathment. The agent is no longer optimizing her personal consumption preferences over an infinite horizon, which results in increasing the optimal consumption-income ratios during the agent's lifetime, even in the risk-free economy. The size of the planned bequeathment also impacts on the optimal consumption-income ratios, as expectations of the terminal consumption level must be quantified, with the obvious result that the higher the bequeathment, the greater the savings rate, given an asset rate of return. However, only risk-free asset returns *implies the per capita consumption growth rates* that would be indicated in the Mehra-Prescott infinite horizon optimality conditions.

What is different in the risk-free economy for an agent with wealth preferences is the amount of savings that accumulates, and more importantly, a higher aggregate utility is much of personal consumption, not to include any externality from bequeathment. Therefore, the un-calibrated wealth preferences model adjusts the initial consumption-wealth ratio (consumption-income ratio in a risk-free economy with positive returns) to a level necessary to ensure the consumption growth rate expected in the equity premium puzzle, conditional on the US average for asset rates of return.

To account for expectations of both a terminal time period and the amount of bequeathment, I assume a wealth process that sets the target level of wealth at a finite time (T). The agent preferences over consumption are log utility up to T, and then I assume a higher CRRA value to represent the agent's utility from leaving an inheritance, which is an important point of departure from the Mehra-Prescott assumption. Otherwise, there is no reason for an agent not to follow the consumption process of an infinitely-lived agent, as the finitely-lived agent simply bequeaths the level of wealth at time T and the other continues consuming a portion of the dividend. Generally, *wealth preferences* will imply a smaller bequeathment than the infinite horizon Mehra-Prescott model.[9]

[9] Merton (1973) develops an intertemporal capital asset pricing model for multiple agents, each living a number of years represented by random variable T and valuing a terminal wealth function. While the purpose of his model is to assess optimal portfolio selection when there are changing investment opportunity sets, Merton notes that the assumption of log utility collapses his results back to the classic CAPM. Since the two-fund separation theorem may

Additionally, the variable (*i*) in the wealth constraint determines the expected growth rate of wealth and is necessary to define expectations of terminal consumption, which could be any amount, to include consuming all wealth accumulated, as I constrain borrowing. Therefore, for one special case of (*i*), the infinite horizon conditions are optimal. I choose to assume agents preserve wealth, in part because the generality of the model allows for the endowment to be the implied present value of human capital, so that an agent could have virtually nothing initially, receive dividends on her human capital endowment in the form of personal income and then expire without leaving any savings, but also without debt.

The combination of low consumption growth rates and relatively high consumption-income ratios require relatively high asset rates of return that are comparable with the historical averages of risky assets in the Mehra and Prescott data set.[10] For example, risk-free rates of 10% (7%) generate real consumption growth rates of approximately 3% (2%), even though the initial consumption-income ratio for the 20-year old agent is only 70% and the agent assumes she will live till 80 years old and die with zero change in net savings. If the agent assumes a shorter lifespan or lowers her initial consumption-income ratio, the optimal consumption growth rate increases. Note that agents who choose to save at a rate that is consistent with an infinite horizon do not generate a puzzle as to the equity Euler condition in the Mehra-Prescott model.

Importantly, the initial consumption-income ratios in Tables 4-6 are exogenously determined and lower than NIPA data reports. In this respect, the wealth preference model is empirically emulating the US economy, without fully justifying 70% or higher consumption income ratios, especially since this paper assumes only risk-free returns. For example, from 1889-1978, approximately one-third of the rates of return on equity in the Mehra-Prescott data set are less than one percent, about one-third are between one and ten percent, and the remainder are greater than ten percent. Calculating a rough estimate of the expected initial consumption-income ratios using these weights put the initial ratio at over fifty percent, which is a significant increase from the ten to twenty percent implied if the agent behaves as though she has an infinite horizon. The impact of choosing an expected initial consumption-income ratio from these three possible states is to lower the Mehra-Prescott consumption growth from the 5.9% rate that is consistent

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not be true when preferences over consumption bundles are non-homothetic with respect to income, the paper is largely uncorrelated with the analysis here.

[10] Mehra-Prescott assume expected utility, so certainty equivalents are appropriate in any case, but I will derive upper bounds on consumption growth rates for log utility preferences of 3% or less, even for average rates of real return that significantly exceed the average rate of return in the Mehra-Prescott data sample.

with a real return of 7%, to nearly 3.5%, while the consumption growth rate in a risk-free economy that returns 3.5% is 2.5% (see Table 1). Essentially, the higher (expected) initial consumption income ratio determines a path that lowers the consumption growth to a rate that satisfies the Mehra-Prescott Euler condition in a risk-free economy if the asset rate of return is only 4.5%, not 7%.[11]

Insert Table 1.

Additional insight into consumption-income behavior in times of declining markets may increase the initial percentage. Also, a model that seeks to fully endogenize the eighty percent average ratio may require evaluating other consumer behavioral factors, or it may even be true that average wages are somewhat Malthusian. In any case, wealth preferences represent a significant step in this pursuit, as well as providing a straightforward method of calibrating consumption-income ratios and assessing their impact on consumption growth rates. Essentially, the higher the consumption-income ratio, the lower the consumption growth rate if given the initial consumption income ratio, so using ratios in Tables 5, 6 and 7 lower than the 80% US average strengthens the analysis.

Historical population data from the Census Bureau provides much of the intuition for a model of wealth preferences. A very small percentage of the population lived past 65 during the first half of the sample period, the percentage increasing from under three percent in 1860 to five percent in 1930. However, by 1950 the 65 and older cohort accounted for 8% of the population and that age group is currently estimated at over 12% of the national total. Therefore, if an agent started work at age 16, then the agent could expect to have only about 50 years of earnings and consumption before death, so that a higher consumption-income ratio might be preferable to the miserly lifestyle indicated by the infinite horizon Mehra-Prescott optimality conditions.[12] Following is an example comparing the consumption process of an agent with wealth preferences with that of an agent who adheres to the infinite horizon assumptions.[13]

Assume that agent *X* starts working at age 20 (or invests her endowment) and receives annually a 5.7% dividend (income) on her perceived present value. Further assume that she initiates and maintains

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[11] The argument can be made that in the risk-free economy, the asset rate of return roughly conforms to an average risky rate of return, in that the risk-free consumption growth rate is a proxy for the average consumption growth rate.
[12] An agent can now be expected to place a greater emphasis on retirement wealth because of increased life expectancy and the market reflects such preferences.
[13] The examples allow the agents to increase the proportion of income consumed until 100%, holding wealth constant afterward. The results of the model optimize from an initial consumption-income ratio, rather than assuming agents readjust their horizon as they age. This is more likely to capture the average consumption growth rate, whereas the real per capita consumption growth rate may actually be nearly zero after retirement for a number of agents.

consumption at 18.6% of the dividend and invests the remainder at the constant, risk-free rate of 5.7%, so that her consumption growth rate is 4.6% *ad infinitum*. In other words, for log utility preferences over consumption and with a time discount of one percent per year, there is no equity premium puzzle with respect to agent *X*, as *X's* choices satisfy the (only) optimality condition in the model.

Allow that at the same time and age as *X*, agent *Y* initiates consumption at 70.0% of her initial income, invests the remainder in the same risk-free economy and increases consumption at only 2.0% per year until age 65, at which time *Y* is consuming approximately 100% of the dividend. The average consumption-income ratio throughout *Y*'s adult life is 80% or higher, a percentage that is consistent, if we use recent population distribution estimates, with national income and product account (NIPA) data from 1927 to 2007. Naturally, the accumulator utility function of *X* will surpass that of *Y*, and, in this example, that does not occur until *X* and *Y* are 124 years old, even if *Y*'s consumption after 65 is held constant for the remainder of her life. At age 65, *X*'s wealth from the asset is seven to eight times the original investment, while *Y*'s wealth is nearly twice that of the initial capital outlay. [14]

One concern that may arise because of the approach I use is that Mehra and Prescott calibrate consumption growth rates in order to observe the dynamics of asset rates of return. However, in a risk-free economy, the inverse functional relationship holds so that I could assume low consumption growth rates to generate the required asset rates of return. Essentially, if I assume 3% consumption growth rate and the initial consumption-income ratio is 70%, etc., I will find that 10% is the required asset rate of return to support the growth rate of 3%.

Another concern could be that personal income is not accounted for or that many in the US do not operate in any real sense from an endowment. The model is very general, so initial wealth (endowment) may simply be the implied present value of human capital (labor, entrepreneurs) and those with financial endowments could generally be capitalists, although this is obviously an oversimplification. In the former case, the dividend becomes personal income, so that all revenues are accounted for in the aggregate representation.

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[14] The return rate of 5.7% is the log utility certainty equivalent for the actual equity returns of the 1889-1978 Mehra-Prescott data, assuming that $1/n$ is the relevant probability for each of the n data points. The average of those real equity returns is almost 7%. Using the higher return rate (7%) in the risk-free economy, *X* initiates and maintains consumption at 15.3% of income. If *Y* initiates at 73.8% and maintains a constant growth rate of 2.0%, then she is consuming nearly 100% of income at age 65. In this scenario and again holding *Y*'s consumption level constant after 65, the accumulator of utility for *X* surpasses *Y*'s when they are both 109 years old.

Yet another concern is that a risk-free rate of return clearly focuses on the level of return and not the (lack of) volatility in the consumption growth rate, or conversely, the relatively high volatility in equity returns, conditional on the observable consumption process. The literature that emphasizes the role of volatility in the puzzle uses a simple statistical relationship to decompose the expected value of the product of two random variables and then offers evidence that the problem area is primarily the covariance of equity rates of return with inverse of consumption growth rates: $\text{Cov}(x,y) + \text{E}x\text{E}y = \text{E}xy$. The statistical relationship holds reasonably well for CRRA values that satisfy the equity Euler condition (between 2.5 and 5). I also show in that range of CRRA values that the *increase in the implied covariance of consumption growth rates and real equity rates of return are virtually immaterial* compared to the impact of the increase in the implied growth rate of consumption. However, to be thorough, I show conditions in the appendix under which this model implies relatively smooth consumption growth rates when there is risk from multiple states of nature.

Additionally, the fact that the model here defines wealth preferences as the amount of wealth expected at death, or more generally at a given point in the future, the literature on the role of savings in the equity premium puzzle may appear to have already covered this ground. However, I generalize the wealth process so that an agent could consume all their wealth at the assumed time of departure, have just preserved their wealth, or have saved as much as desired within feasible constraints. In the paper, I assume that agents only just preserve the initial level of (implied) wealth, so that net savings growth is zero, and yet I still generate low consumption growth rates. Therefore, the impact of a positive net savings is a lowering of the optimal consumption growth rate from the estimates I provide, but this is an adjustment from a growth rate that is already in the 2%-3% range. Note that there would not necessarily need to be any change in the asset rate of return, simply a lower consumption growth rate up to the assumed terminal year.

Finally, I do not attempt to explain the US average risk-free rate in the context of growth rates of non-durables and services, because Kuhn-Tucker tests of the optimality conditions show that values of CRRA that are high enough to equate the sample estimates of equity Euler equation with the sample estimates of the risk-free Euler equation tend to invalidate each individual optimality condition. Additionally, McGrattan and Prescott (2003) argue that the consumption of non-durables and services is usually not funded by the short-term risk-free assets in the Mehra-Prescott model. Therefore, I accept that hypothesis as correct, implying that the average risk-free rate would be higher if competing with equity to fund the same consumption bundles.

The next section addresses the importance of co-volatility of consumption in the puzzle. Section 3 briefly reviews the literature, Section 4 develops the model, and the results are tabled in Section 5. Section 6 is the summary.

2. Equity Premium Puzzle: Consumption Growth Rates or Consumption Smoothness?

Mehra and Prescott find that consumption growth rates are too low and/or too stable relative to the risk premiums implicit in asset rates of returns for predicted levels of CRRA. The relatively low and stable consumption growth rates appear to be precautionary behavior, so that one possible answer to the puzzle is that consumers are more risk averse than originally thought. However, I show that observable consumer behavior is considerably different from the Mehra-Prescott infinite horizon assumptions and different than that predicted by precautionary preferences, as agents tend to consume such a large proportion of income as to restrict consumption growth to a rate well below that of the average rate of return on equity.

To show that the smoothness of consumption with respect to real equity returns is not the main issue of the puzzle, in Table 1 are the sample means and covariances of the Mehra-Prescott equity optimality condition for several CRRA values ranging from 1 to 45. The equity Euler equation for the puzzle is well-known and derived more formally in the model section of the paper. The version of the equity Euler equation presented in the next paragraph is useful in clarifying a common misconception in the literature, which is that the relatively low co-volatility of the consumption growth rate is at the heart of the equity premium puzzle. Instead, the relatively low sample average of the consumption growth rate is the key, as Tables 2, 3 and 4 shows.

Let δ = the CRRA parameter, g = growth rate of consumption, R = 1 + rate of return on equity and β = the time discount factor for the next period. Then $\frac{1}{\beta} = E(1+g)^{-\delta}R$ is the equity Euler equation and solves for a CRRA value in the interval between the values 2.5 and 5.[15] Any CRRA-induced adjustments

[15] CRRA values that set the "right-hand side" of the equity Euler equation equal to the "right-hand side" of the risk-free Euler equation are typically measured as well over 20, with estimates of 35 or higher common in the literature. However, it is straight-forward to show that those high CRRA values invalidate the "right-hand side" of each Euler equation, so that such solutions satisfy necessary, but not sufficient conditions for an optimum.

to the right-hand side of the equity Euler equation can be related to the changes necessary to the consumption growth rate (g*) for log utility to be the correct preferences: $E(1+g)^{-\delta}R \equiv E(1+g^{*})R$.

Without loss of generality, I set β =1 and find that increasing the CRRA parameter in the aforementioned identity affects the sample mean of g * considerably more than the variance for values of δ that solve the equity Euler equation. Note that columns 6 and 7 in Table 2 show that for CRRA values less than 10 the sample average for the product of two random variables is approximately equal to the sum of the sample covariance and product of the sample means. For values greater than 10, the discrepancy in the values in columns 6 and 7 imply that the aforementioned statistical relationship does not hold for the data at those levels of risk aversion, so that arguments about the role of covariances and means in the equity premium puzzle are much more difficult to empirically validate for high CRRA values. Note that the underlined values in columns 6 and 7 indicate that the aforementioned statistical relationship holds reasonably well for CRRA values in the interval (2.5, 5).

Table 2 shows that an increase in the mean of g* is the major change that sets the right-hand side (RHS) of the equity Euler equation equal to the left-hand side: $\frac{1}{\beta}$. Although the sample covariance between g* and R also increases, the level of increase is negligible when compared to the change in the sample mean.

Insert Table 2.

Table 3 clarifies the results in Table 2, listing the mean and covariance (with real equity returns) of the implied per capita consumption growth rate (g*) for several CRRA values between 1 and 45. Although the sample variance of the consumption growth rate increases, it is the increases in the sample average that are the most significant until CRRA values of over 30 are sampled. However, as noted above, *assuming a CRRA value greater than 5 will result in sub-optimal estimates of the RHS of both the equity and the risk-free Euler equations*. Instead, models purporting to resolve the puzzle with such high CRRA values are simply equating the RHS of the equity Euler equation with the RHS of the risk-free Euler at some value considerably less than β^{-1}.

Insert Table 3.

The sample averages below in Table 4 present the theme of the paper: the equity Euler equation can be deemed optimal for log utility by Mehra-Prescott standards given the equity rates of return,[16] if there is a location shift in the sample distribution of the annual real consumption growth rate from 1889-1978.

Insert Table 4.

This section presents evidence that the level of the consumption growth rate is the critical element in satisfying the equity Euler equation from the puzzle. Regardless of this assertion, in the interest of clarity the section developing the model offers a proof that wealth preferences, even for zero net savings over a lifetime, implies lower consumption co-volatility for expected utility than the Mehra-Prescott optimality conditions.

3. **Literature Review**

Lucas (1978) proves general conditions under which there is a "fixed point" policy process and Mehra and Prescott (1985) test the Lucas model for annual rates of asset returns and consumption growth in the special case of CRRA preferences. Assuming that both the risk-free and market returns fund the aggregate consumption bundle of non-durables and services, Mehra and Prescott find that the joint distribution of consumption and asset returns implies that CRRA values over thirty are necessary to emulate empirically observable risk premiums, while the literature they reference indicates that CRRA values between one and two are appropriate.[17] There are now hundreds of papers addressing the equity premium puzzle.[18]

Descriptively, the research on the puzzle runs the gamut from accepting very high rates of risk aversion as correct to increasing the degrees of freedom for the CRRA parameter estimate by increasing the number of parameters in a consumption-savings model. Intuitively, many of the arguments include some kind of precautionary motive to explain the combination of relatively low consumption growth rates and relatively high asset rates of return, but also there are models that separate intertemporal risk aversion from the diminishing marginal utility of wealth, as well as those matching asset returns with specific consumption bundles and specific consumers.[19]

[16] The value 1.000604 in Table 3 is close enough to 1.0 to validate that optimality condition.
[17] Lucas (2003) sets the long-run upper bound for CRRA at 2.5.
[18] Mehra and Prescott (2003), Siegel and Thaler (1997) and Kocherlakota (1996) provide reviews of various approaches to solving the equity premium puzzle.
[19] By retaining the additively-separable power utility function from Mehra-Prescott, this paper shows that separating the elasticity of substitution from diminishing marginal utility is not necessary to explaining the high assets rates of return concurrent with relatively low consumption growth rates.

The Role of Wealth Preferences

The idea of wealth-related constraints is not new in the literature on the equity premium puzzle, but the theme of explaining the optimal rates of change in relation to the initial consumption-wealth ratio is new. For example, overlapping generation (OLG) models constrain wealth and explain the puzzle, of which Constantinides, Donaldson and Mehra (2002) may be considered the example. The basic idea for a two-generation model is to assume that agents consume 100 units while young and 110 units when old, so that the evenly weighted average of overlapping consumption is always 210/210, even though each individual agent is increasing consumption at 10% between generations. Slight modifications or extensions of this simple framework can result in a positive but constrained growth rate of aggregate consumption. In the case of the three-generation Constantinides model, the junior cohort can't borrow, and only the middle-aged cohort purchases assets. Still, this assumes that middle-aged agents are typically able to increase consumption at the rates indicated by Mehra-Prescott, a questionable assumption given the empirically high consumption-to-income ratios. I do not use the OLG framework, as the additional CRRA parameter in my model affects only the amount of inheritance. Therefore, there is only one CRRA parameter explaining consumption behavior.

Precautionary motives imply relatively low consumption-to-wealth ratios. Gourinchas and Parker (2001) and Parker and Preston (2005) assume precautionary savings, as do Gourinchas and Parker (2002), who estimate changes in consumption preferences over the life cycle. I also assume that risk aversion preferences are not identical over an infinite horizon, but only with respect to leaving an inheritance. Other examples of research that assume precautionary motives to account for the relatively low consumption growth rates include Rietz (1988) and Barro (2006), who model the after-effects of economic disasters. Additionally, Heaton and Lucas (1996) constrain borrowing in order to smooth income uncertainty.

A framework for a higher initial ratio of consumption to wealth might have been found in the habit formation models of Constantinides (1990), Abel (1990) and Campbell and Cochrane (1999). For example, there may be an element of behavior similar to Abel's "keeping up with the Joneses" that helps to explain the relatively high ratio of consumption to wealth for younger agents. However, the focus of habit formation is on constraining optimal consumption growth rates, so that the initial ratio of consumption to wealth is accepted without explanation. If an agent did start her consumption path at the low level implied by the Mehra-Prescott model, imposing habit formation preferences could result in a miser's consumption-wealth process. Note that in my model the higher initial consumption-income levels are assumed to be optimal even if an agent is alone on an island (e.g. a Robinson Crusoe economy).

Ait-Sahalia, Parker and Yogo (2004) match asset returns to consumption preferences of wealthier agents and estimate a significantly lower risk premium for luxury goods. McGrattan and Prescott (2003) assert that short-term risk-free rates do not usually fund non-durables and services, which questions the assumption that an aggregate good is also the representative good, as the market does not "clear" with respect to non-durables and services and risk-free rates.[20] Lustig and Van Nieuwerburgh (2005) and Piazzesi et al (2007) note that aggregates of durable goods consumption may be more sensitive to asset rates of return than non-durables and services.

Yogo (2006) does not model a representative consumption good or a representative risky asset rate of return. Instead, he assesses the intratemporal and intertemporal conditions and argues that high levels of risk aversion are a necessary condition to explain observable consumption growth rates and asset returns, but in this paper I contradict that assertion. To summarize, all of the aforementioned models are unable to empirically support the assumption of log utility preferences.[21]

4. **Model**

The ideas for the model are simple: economic agents know that they will not live forever and that the utility of wealth that is bequeathed may be different than when the same level of wealth is used for their own consumption. Of course, diverse wealth preferences can be seen, and many agents do not have much in the way of an endowment other than human capital. The model here is designed more to reflect the behavior of an aggregate agent than to be a strict representation of identical preferences.

Formally, I generalize an infinite horizon model such that there is a reasonable expectation that death will occur at some $\tau \geq T$. The value of T is known to the agent at the time of the initial consumption-savings decision, such that the agent stabilizes the consumption growth rate over the projected lifetime.

The result of assessing the initial consumption-income ratio and the optimal expected consumption growth rate based on T is that at each time period the accumulated utility is optimal, conditional on the uncertainty as to when the period of personal consumption will end. The lower marginal utility of leaving

[20] Kuhn-Tucker tests can determine whether strictly interior solutions are optimal.
[21] Recently, David (2008) achieves low CRRA values by modeling speculative, competing agents who inherently provide systematic deviations from a classic rational expectations framework.

an inheritance is represented in the model by a higher CRRA value, but in the model the agent's CRRA preferences for their personal consumption do not change. The anticipated level of bequeathment is the original level of wealth, so there is only one CRRA parameter to calibrate concerning the consumption growth rate.[22]

The last subsection looks at conditions that imply smoother consumption growth rates than anticipated from the Mehra-Prescott optimality conditions. Most of the detail is referenced to the appendix, but the methodology is to implicitly differentiate equation (4) or (5), find the elasticity and then assess the points where the elasticity of the consumption growth rate with respect to a change in the risk-less rate of return is positive, but less than unity. Smoother consumption paths for expected utility are then assured by a convex combination of such points satisfying the criteria. Wealth preferences and zero net savings require risk in asset returns to endogenously restrict the consumption-income ratio sufficiently to emulate the US data, so I simply calibrate the model in the empirical section with initial consumption-income ratios.

4.1 Equity Premium Model

First, I develop the Euler equations of the equity premium puzzle to highlight differences in the model of wealth preferences. Consumption c_t, and the prices p_t^k, dividends d_t^k and shares s_t^k of asset class *k* are real-valued, non-negative processes in the rational-expectations, expected-utility model and are chosen from convex sets that are compact at any time *t* and of bounded variation $\forall t$.[23] The following model reflects the agent's preferences and budget constraint.

$$max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \colon U(c_t) = \frac{c_t^{1-\delta}}{1-\delta}, 0 < \beta < 1, 0 < \delta$$

$$s.t. \;\; c_t + s_{t+1}^k p_{t+1}^k \leq s_t^k (p_t^k + div_t^k) \colon k \in \{1, \dots n\}$$

Optimizing consumption funded by risky and risk-free asset classes with respect to the number of shares for the next period s_{t+1}^k results in familiar Euler equations of the equity premium puzzle in equations (1) and (2). Note below that $C_{t+1} = \frac{c_{t+1}}{c_t}$ and $R_{t+1}^k = \frac{P_{t+1}^k + div_{t+1}^k}{p_{t+1}^k} \colon \; EC, ER < \infty$:

$$1 = \beta E_t C_{t+1}^{-\delta} R_{t+1}^{equity} \tag{1}$$

[22] In reality, an agent may change preferences, for example after retirement or if living longer than expected.

[23] i.e. Lebesgue measurable. A proof of a non-measureable Lebesgue set is equivalent to asking how many zeros must be added together so that the sum is a positive value, which does not exist in economics or finance and therefore there are no set structures to exclude. Additionally, the assumption of rational expectations bounds the probability measure and the continuous mapping is from sets of real numbers to real numbers at a level of refinement such that a function exists

$$1 = \beta E_t C_{t+1}^{-\delta} R_{t+1}^{risk-free} \tag{2}$$

4.2 Model of Wealth Preferences

In this section, I replace the shares and prices in the budget constraint of the equity premium model with a wealth variable: $w_t \stackrel{\text{def}}{=} s_t p_t$. This substitution in itself has no material effect on equations (1) and (2), but the wealth variable (i.e. *wealth base*) is more intuitive for understanding the accumulator wealth function. Also, I assume a change in the marginal utility of wealth, which is reflected in preferences over the time *T* claim on future consumption.

$$max E_0 \sum_{t=0}^{T-1} \beta^t \frac{c_t^{1-\delta}}{1-\delta} + \sum_{t=T}^{\infty} \beta^t \frac{c_t^{1-\lambda}}{1-\lambda}, \ 0 < \beta < 1, 0 < \delta, \lambda$$

$$s.t. \ c_t + w_{t+1}^k \leq w_t^k (1 + d_t^k): d_t^k \stackrel{\text{def}}{=} \frac{div_t^k}{w_t^k}$$

$$s.t. \ w_T^k = w_t^k (1+i)^{T-t+1}: -1 \leq i \leq R \stackrel{\text{def}}{=} 1 + d_t^k$$

The Mehra-Prescott assumption is $\delta = \lambda$. For example, if $\lambda \rightarrow \infty$, then the agent expects to remain at a constant level of wealth after time *T* by consuming the entire dividend, preserving the level of wealth and only consuming the dividend. The wealth constraint is necessary because of the finite horizon, which requires a decision about the expected optimal allocation of time *T* wealth into consumption and bequeathment. Note that the constraint allows for flat or negative net returns on wealth and is likely to emulate the actual decision process of the agent: $w_t^k(1 + i) = w_{t+1}^k$.

I choose to model one wealth process, assume $\delta < \lambda$ and expect that $\delta = 1$. If *i* = -1, then $w_T = 0$, and if *i* = 0, then $w_T = w_t$, which is a self-financing condition. In a closed economy, *i* = -1 serves as a lower bound, because net borrowing is zero. The general wealth process is given below, with c_t the initial consumption level and assuming consumption from the dividend at time *t*. The assumption is that the variable w_t is the wealth base at time *t*, so the agent's total return when choosing c_t is $w_t(1 + r_j)$, so that one index upper limit is *T*+1.

$$w_T = w_t \prod_{i=t}^{T+1}(1 + r_i) - \sum_{i=t}^{T} c_i \prod_{j=i+1}^{T}(1 + r_j) \tag{3}$$

My model economy has one asset class funding the consumption bundle, and *i* = 0. I further restrict the economy so that the value assumed in each state for the asset rates of return, consumption growth rates, and wealth growth rates are invariant with respect to time. This simplification will make it easier in the next section to assess limit properties of the function, as the second RHS term in (3) then reduces to a form of the Gordon growth model. Setting the invariant version of (3) equal to the wealth constraint in the model determines the optimal consumption growth rates for a given level asset rate of return and wealth process, or conversely, solves for the asset rate of return if given the consumption growth rate. Assuming log utility and invariant returns in (3) implies the expected consumption process in (4): $w_t R$ is total wealth available for consumption at time (t).

$$c_t(T) = w_t E_t[R^{T-t} - (1+i)^{T-t}](R^{T-t+1} - C^{T-t+1})^{-1}(R - C) \tag{4}$$

Note that the time preference (β) is implied through the variable i, and that C is consistent with equations (1) or (2). Additionally, the higher the savings rate (i) is, the lower the initial consumption-wealth (consumption-income in these simple economies). The *savings rate is an additional constraint on the growth rate when calibrating the model with observable consumption-income ratios*, as the left-hand side of equation (4) then constrains C for a given R, and increasing the savings rate will constrain C even more.

4.3 Initial Consumption Level

Next, the optimal initial consumption levels for limits of the new preference assumptions are derived by taking limits of the optimal initial consumption level function.[24] The main purpose is to show that if $T \to \infty$, the optimal initial consumption value and therefore the optimal consumption path for each level of wealth, is much lower than intuition or observable behavior indicates. For example, if initial wealth is 1000 units, the risk-free return is 10%, and the time preference is 1%, then the Mehra-Prescott model would expect initial consumption of just over 10 units for log utility preferences. If the agent started consumption at age 20, she would not consume as much as 80 units in any given year until in her 40s.

Consumption decisions in period (t) are the result of investment decisions in period (t-1). The limit for initial consumption as $T \to t$ is $w_t\big(R - (1+i)\big)$, so if $i = -1$, the agent consumes everything, remembering $w_t R$ is time (t) total wealth by virtue of the choice of indices. The one period choice (t+1) is:

$$\lim_{T\to t+1} c_t(T) = \lim_{T\to t+1} w_t E_t[R^{T-t+1} - (1+i)^{T-t+1}](R^{T-t+1} - C^{T-t+1})^{-1}(R - C) \tag{a1}$$

$$\lim_{T\to t+1} c_t(T) = w_t E_t\,(R^2 - (1+i)^2)(R^2 - C^2)^{-1}(R - C) \tag{a2}$$

[24] The expected wealth at time ($t + n$) is constrained by the expected asset rate of return raised to the nth power and by initial wealth, so the exchange of limits is permissible by the Lebesgue dominated convergence theorem.

Testing the limit in (a2) for a condition of no savings (i = -1) results in $c_t \approx \frac{1}{2} w_t E_t R$: the agent will consume everything in the two periods. If $(1+i), C < R$, then the infinite horizon consumption-wealth ratio is below in (a3):

$$\lim_{T\to\infty} c_t(T) = w_t E_t (R - C) \tag{a3}$$

4.4 Solving for the optimal rate of consumption growth for $c_t^*(T)$

If calibrating an initial consumption value $c_t^*(T)$, then equation (5) recovers C^*, which is the per capita consumption growth rate that results in the initial investment as the level of wealth at T when consumption starts at $c_t^*(T)$. Calibrating this equation produces the tables in the next section.

$$0 = w_t E_t \left[R^{T^*-t} - (1+i)^{T^*-t} - \frac{c_t^*(T)}{w_t} \right] \left(R^{T^*-t+1} - C^{T^*-t+1} \right) (R - C^*)^{-1} \tag{5}$$

The average rate of wealth increase in equation (6) derives from equation (4) using the fact that for a risk-free return $C = \gamma R, 0 < \gamma < 1$, and $\gamma = \beta$ if equations (1) and (2) hold. The variable *inc* is income.

$$(1+i) = \left(R^{T-t+1} - \frac{c_t}{inc_t} \frac{inc_t}{w_t} \left(\frac{1-\gamma^{T-t+1}}{1-\gamma} \right) \right)^{\frac{1}{T-t+1}} \tag{6}$$

If modeling using calibrations, then $\gamma = \frac{C^*}{R}$. In equation (7), the return is certain and can vary with each time period, so that (7) is a rough estimate of one of four values in a risky market, each conditional on the other three: consumption-wealth ratio at time t, desired wealth at time T, consumption growth rate or required rate of return ($\gamma^*$is the geometric mean).

$$c_t(T) = w_t \left(1 - \prod_{k=t}^{T-t} \frac{1+i_k}{R_k} \right) (1 - \gamma^*) (1 - \prod_{k=t}^{T-t+1} \gamma_k)^{-1} \tag{7}$$

4.5 Lower consumption co-volatility for wealth preferences

The criteria establishing the relative consumption co-volatility for wealth preferences when compared with the Mehra-Prescott model, relies on the fact that expected utility is a weighted average of certain outcomes and are in the appendix. Implicitly differentiating the Mehra-Prescott equity Euler equation quickly reveals the instantaneous rate of change of consumption for an equivalent rate of change in the asset return is C/R. Implicit differentiation of equation (4) or (5) identifies (T, i, γ) sets of points where co-volatility is lower than implicit in the Mehra-Prescott optimality conditions, given rates of return and time preference.

$$i_{\frac{\partial C}{\partial R}=0} = -1 + R\left[\frac{1}{\gamma} + \frac{\left(1-\frac{1}{\gamma}\right)}{(T-t+1)^{-1} - \left(\frac{1-\gamma}{(1-\gamma^{T-t+1})}\right)}\right]^{\frac{1}{T-t+1}} \tag{8}$$

In equation (8), setting the elasticity to zero solves for the (i,T) set of points that serve as the boundary between negative and positive elasticity, holding constant R and γ: $C = \gamma R$, $0 < \gamma < 1$ and $\gamma = \beta$ in the table. If $T \to \infty$ in (8), then (1+ i) equals R, implying that a constant consumption growth rate in an infinite horizon model for log utility preferences is a path of zero consumption.

5. **Risk-Free Economy**

In this section, the tables show the consumption growth rate for a twenty-year old agent who will retain just the initial investment at age 70 or age 80. A larger accumulation of wealth could be assumed, but that would only serve to further restrict the consumption growth rate. Therefore, the values in the tables serve as a somewhat conservative estimate of the impact on the consumption growth rate of high consumption-income ratios.

Two risk-free economies are calibrated; one with the rate of return for the asset equal to 10% and the other 6.98%, proxies for the average nominal and real rates of return for equity in the 1889-1978 data sample from Mehra and Prescott. Using these high rates of return in equations (4) and (5) of the previous section clearly shows that consumption growth is constrained to rates well below those predicted in the Mehra-Prescott model, but these same rates are reasonably consistent with observation. Any value of time (t) wealth is appropriate, as equation (4) shows that the model actually solves for an initial consumption-wealth *ratio*. The risk-free assumption simplifies the argument for replacing consumption-wealth ratios with consumption-income ratios, which NIPA typically records at 80% or greater.

Tables 4-6 provide examples of optimal consumption ratios from equation (5) that are conditional on equation (3).[25] Allowing that the right-hand side of (5) is $f(C_t^*)$, the Newton-Raphson method generates the values of C_t^* in Tables 4-6: $C_{t+1}^* = C_t^* - \frac{f}{f'(C_t^*)}$.[26]

[25] The initial consumption-savings ratio serves the same purpose as an initial consumption-wealth ratio in an economy with only a risk-free asset, such that $\beta R > 1$.
[26] The iterations are sensitive to the initial choice of C^*, but simply iterating the initial choice quickly bounds the value of C^* that will solve equation (6).

The far left-hand column of Table 5 shows that a 20-year old who consumes 75% of the initial dividend *will just maintain initial wealth by age 80*, if consumption increases at a rate far less (1.8%) than indicated by the Mehra-Prescott model (8.9%).

Insert Table 5.

Insert Table 6.

Insert Table 7.

Tables 5, 6 and 7 show that even if planning to retain only the level of initial investment as wealth, following a path of high consumption-income ratios serves to restrict per capita consumption growth to rates well below those forecasted in the optimality conditions of the equity premium puzzle. In these tables, the consumption growth rate is calculated that will leave only the initial level of wealth at time *T*. Clearly, as is true in the example in the introduction, if an agent's wealth preferences includes leaving a much larger inheritance within the framework of high consumption-income ratios, then the inheritance will further restrict consumption growth rates, but such assumptions are empirically consistent with precautionary savings and are not necessary to explain the equity Euler equation puzzle. Note that the example in the introduction emphasizes that the accumulator utility function for an agent that follows the Mehra-Prescott consumption-income path and consumption growth rate will eventually surpass that of an agent who conforms to observation, but depending on the asset rate of return, time discount and consumption-income path, the agent may need to live well past 100 years to do so.

6. **Concluding Remarks**

If *T*, the random variable reflecting an agent’s expectations of demise, approaches infinity, then the results here will conform to the Mehra-Prescott model. In a finite time horizon, a wealth constraint determines the expected disposition of terminal wealth, the possibilities range from consuming all remaining wealth to bequeathing the wealth, with the most likely pattern some factor of the (implied) initial endowment level is preserved and the agent consumes the dividend in the event that she outlives her own expectations. Wealth preferences indicate that agents’ consume a much higher proportion of their dividend (income) during their expected lifetime than they would if following the path indicated by an infinitely-lived agent, with the result that high asset rates of return are concurrent with relatively low per capita consumption growth rates, the puzzle of the equity Euler equation in the equity premium puzzle.

Calibrating the wealth preference model with the relatively high average consumption-income ratios reported in NIPA is sufficient to restrict the average per capita consumption growth to rates well below those expected in the Mehra-Prescott infinite horizon model. By also restricting the wealth process to zero net savings for the agent's expected lifetime in the empirical section, I show that a precautionary savings strategy does not resolve the puzzle so much as it would further restrict consumption growth rates. Additionally, I show the importance of the level of the consumption growth rate in satisfying the equity Euler equation from the puzzle, rather than the co-volatility of the consumption growth rate with asset rates of return

The Mehra-Prescott model is then a special case of wealth preferences when an agent's utility from personal consumption equals the utility from leaving an inheritance, which implies a savings-income ratio of 80% or higher for US averages of equity return rates over the last century. The de-emphasis on the volatility of the consumption growth rate and asset rate of return facilitates both evaluating the initial consumption-wealth ratio, which translates simply into evaluating the initial consumption-income ratio.

The risk-free rate of return in the sample economies is either seven or ten percent, proxies for the average real and nominal risky return of the 1889-1978 Mehra-Prescott data. The risk-free economy can be replaced with an economy that has a risky asset where the aggregate agent seeks to preserve an average level of wealth, conditional on initiating consumption at levels consistent with NIPA data. However, using risk-free rates increases the per capita consumption growth rate in the results section, so that the estimates serve as upper bounds and are still very much in line with observation. Empirically, consumption-income ratios of seventy percent and higher are normal, and ultimately these ratios are too high to support the rates of consumption growth implied by Mehra-Prescott optimality conditions for log utility without eventually reducing the initial investment base.

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APPENDIX

The Mehra-Prescott optimality conditions: implicit differentiation of equation (2) implies

$$\frac{\partial C}{\partial R} = \frac{C}{R} \tag{A1}$$

Quickly, it can be seen that the elasticity of consumption with respect to asset returns is unity.

For the model of wealth preferences, (A2) shows the implicit differentiation of equation (4):

$$\frac{\partial C}{\partial R} = A + B \tag{A2}$$

$$A \stackrel{\text{def}}{=} \frac{(T-t+1)R^{T-t}(R-C)\left(R^{T-t+1}-C^{T-t+1}\right)}{\left(R^{T-t+1}-(1+i)^{T-t+1}\right)\left[\left(R^{T-t+1}-C^{T-t+1}\right)-(R-C)(T-t+1)C^{T-t}\right]}$$

$$B \stackrel{\text{def}}{=} \frac{\left(R^{T-t+1}-C^{T-t+1}\right)-(R-C)(T-t+1)R^{T-t}}{R^{T-t+1}-C^{T-t+1}-(R-C)(T-t+1)C^{T-t}}$$

Noting that C, $(1+i) < R$, indexing the level of wealth in time (t) so that the first period constraint is $w - c$ instead of $wR - c$ and after some algebra, then (A3) confirms the limit as T approaches infinity of the wealth preferences derivative of consumption with respect to the asset return is equal to the Mehra-Prescott optimality conditions.

$$\lim_{T\to\infty} A + B = \frac{C}{R} \tag{A3}$$

The task is now to establish criteria under which the elasticity of consumption growth rate with respect to changes in the risk-less asset rate of return is less than unity, but positive. This is equivalent to establishing when $0 < \frac{(A+B)}{\gamma} < 1$. It is true that $A + B > 0$ and A+B <1 if (a4) and (a5) are true:

$$1 - \left(\frac{1}{\left(1-\gamma^{T-t+1}\right)} - \frac{1}{(T-t+1)(1-\gamma)}\right) > 0 \tag{a4}$$

$$1 < (T-t+1)\left(\frac{1-\gamma}{1-\gamma^{T-t+1}}\right)\gamma^{T-t} - \frac{(T-t+1)^{-1}-\left(\frac{1-\gamma}{1-\gamma^{T-t+1}}\right)}{(1-\gamma)} \tag{a5}$$

Table 1. The Optimal Initial Consumption-Income Ratios for a 20-year old Agent. Table 1 shows optimal initial consumption ratios, assuming no net savings, one percent time discount and a sixty time period horizon. If risk indicates that an agent's starts at 54%, even though the average real return is 7%, then the average consumption growth rate will be approximately 3.4% (using equation (5) in Section 4). Therefore, the agent's initial starting consumption-income ratio is consistent for the same horizon as if she is in a risk-free economy that returns 3.5% and her consumption growth rate is only 2.5%, which is optimal by the Mehra-Prescott Euler conditions for a time discount of 1%. Note that 5.9% is the expected growth rate of consumption in an economy of 7% risk-free return, indicating that a risk-adjusted initial consumption-income ratio represents a significant initial step into endogenizing the observably high consumption-income ratios.

Risk-Free Rate	Per Capita Consumption Growth Rate Same as Mehra-Prescott Euler equations (Risk-Free Assets Only)	Initial Consumption-Income Ratio
1.0%	0.0%	98.1%
1.5%	0.5%	85.9%
2.0%	1.0%	75.8%
2.5%	1.5%	67.4%
3.0%	2.0%	60.4%
3.5%	2.5%	54.4%
4.0%	3.0%	49.4%
4.5%	3.5%	45.0%
5.0%	4.0%	41.3%
6.0%	4.9%	35.3%
7.0%	5.9%	30.6%
8.0%	6.9%	27.0%
9.0%	7.9%	24.1%
10.0%	8.9%	21.7%

Table 2. Sample Estimates of the Covariance and Means for Variables in the Equity Euler Equation. If R is the ratio of total return on investment, then the per capita consumption growth rate (g*) for log utility is calculated from the following relation: $E(1+g)^{-\delta}R \equiv E(1+g^{*})R$. Column 6 is the sum of columns 1 and 4, such that the impact of increasing the CRRA parameter on the covariance and on the product of means is aggregated and compared to the actual change that occurs when increasing the CRRA parameter (column 7). The underlined values in columns 6 and 7 correspond to the CRRA interval that satisfies the equity Euler equation.

Column	**1**	**2**	**3**	**4**	**5**	**6**	**7**
	Sample Estimates						
	Cov($(1+g^*)^{-1}$, R)	E$(1+g^*)^{-1}$	ER	E$(1+g^*)^{-1}$ × (ER)		E($(1+g^*)^{-1}$ × R)	
CRRA						**Sum (1+4)**	**Actual**
1	-0.002	0.983	1.070	1.051	β^{-1}	1.05	1.048
1.5	-0.003	0.976	1.070	1.041	β^{-1}	1.04	1.037
2.5	-0.005	0.961	1.070	1.020	β^{-1}	<u>1.023</u>	<u>1.015</u>
5	-0.011	0.931	1.070	0.966	β^{-1}	<u>0.985</u>	<u>0.955</u>
10	-0.022	0.897	1.070	0.846	β^{-1}	0.938	0.824
20	-0.055	0.938	1.070	0.599	β^{-1}	0.949	0.544
30	-0.123	1.176	1.070	0.381	β^{-1}	1.135	0.259
45	-0.430	2.332	1.070	0.160	β^{-1}	2.065	-0.271

Notes: 1889-1978 annual real equity returns.

Table 3. Sample Mean and Covariance of Implied Per Capita Consumption Growth Rate for Several CRRA Values between 1 and 45. If R is the ratio of total return on investment, then consumption growth rate (g*) for log utility is calculated from the following relation: $E(1+g)^{-\delta}R \equiv E(1+g^{*})R$. The underlined values in all three columns correspond to the interval of CRRA values that will solve the equity Euler equation, as per Table 2. This table highlights the significant increase in the sample mean of the consumption growth rate for CRRA values between 2.5 and 5, while the increase in the sample covariance with real equity returns is negligible.

1889-1978 Implied Consumption Growth Rate for Log Utility: g*		
CRRA	**Sample Mean**	**Sample Covariance**
1	1.81%	-0.20%
1.5	3.32%	-0.30%
2.5	6.46%	-0.50%
5	15.08%	-1.10%
10	35.99%	-2.20%
20	98.53%	-5.50%
30	210.34%	-12.30%
45	612.73%	-43.00%

Table 4. Equity Premium for a Location Shift in the Consumption Distribution (μ,σ^2). The *theory* row represents the value each sample average should attain and the (μ,σ^2) row lists the values that result in the equity premium puzzle. The (μ+5%,σ^2) row shows that the equity Euler condition is for all practical purposes optimized if the level of consumption growth rate increases by 5%.

CRRA=1		Euler Conditions		
		equity	***risk-free***	***premium***
Theory		1	1	0
Sample Average	(μ,σ^2)	1.049754	0.991076	0.058678
	(μ+5%,σ^2)	1.000604	0.944569	0.055658

Notes: 1889-1978 annual real equity returns. The time preference is negligible.

Table 5. Model of Wealth Preferences with Differing Risk-Free Rates. The values in the table result from equation (5), which allows for the estimation of the per capita consumption growth rate C^* that will result in a wealth of 1000 units (the initial investment) at time T, conditional on the time discount for consumption (β), total return on investment (R), and initial decision date (t). The first and third columns show that consumption to income ratios of 70% restrict the consumption growth rate to levels far below the return on investment, which is exactly the issue of the equity Euler condition in the puzzle.

$R = 1.1$	$R = 1.1$	$R = 1.0698$	$R = 1.0698$
$\beta = .99$	$\beta = .99$	$\beta = .99$	$\beta = .99$
$C = 1.089$	$C = 1.089$	$C = 1.059$	$C = 1.059$
$w_t = 1000$	$w_t = 1000$	$w_t = 1000$	$w_t = 1000$
$i = 0$ (self-financing)	$i = 0$ (self-financing)	$i = 0$ (self-financing)	$i = 0$ (self-financing)
$T = 80$	$T = 80$	$T = 80$	$T = 80$
$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)
$c_t(T) = 75$	$c_t(T) = 45$	$c_t(T) = 55$	$c_t(T) = 36$
Savings ratio = 25.00%	Savings ratio = 55.00%	Savings ratio = 21.20%	Savings ratio = 48.42%
$C^* \approx 1.018$	$C^* \approx 1.054$	$C^* \approx 1.012$	$C^* \approx 1.036$

Notes: The table shows the results of solving for the implied optimal per capita consumption ratio if an agent expects to live to 80. One economy allows for a risk-free return of 10%, the other matches the 6.98% average of 1889-1978 annual real equity returns. The time preference is one percent.

Table 6. Model of Wealth Preferences with a Larger Time Preference. Increasing the time preference from 1% to 2% slows the per capita consumption growth rate slightly, if comparing the values in Table 6 with those in Table 5. Equation (5) allows for the estimation of the consumption growth rate C^* that will result in a wealth of 1000 units (the initial investment) at time T, conditional on the time discount for consumption (β), total return on investment (R), and initial decision date (t). The first and third columns show that consumption to income ratios of 70% restrict the consumption growth rate to levels far below the return on investment, which is exactly the issue of the equity Euler condition in the puzzle.

$R = 1.1$	$R = 1.1$	$R = 1.0698$	$R = 1.0698$
$\beta = .98$	$\beta = .98$	$\beta = .98$	$\beta = .98$
$C = 1.078$	$C = 1.078$	$C = 1.048$	$C = 1.048$
$w_t = 1000$	$w_t = 1000$	$w_t = 1000$	$w_t = 1000$
$i = 0$ (self-financing)	$i = 0$ (self-financing)	$i = 0$ (self-financing)	$i = 0$ (self-financing)
$T = 80$	$T = 80$	$T = 80$	$T = 80$
$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)
$c_t(T) = 75$	$c_t(T) = 45$	$c_t(T) = 55$	$c_t(T) = 36$
Savings ratio = 25.00%	Savings ratio = 55.00%	Savings ratio = 21.20%	Savings ratio = 48.42%
$C^* \approx 1.018$	$C^* \approx 1.054$	$C^* \approx 1.012$	$C^* \approx 1.036$

Notes: The table replicates the results of Table 5, except that the time preference is two percent instead of one percent.

Table 7. Model of Wealth Preferences with a Lower Life Expectancy. Decreasing the life expectancy from 80 years to 70 years increases the per capita consumption growth rate slightly, if comparing the values in Table 7 with those in Table 5. Equation (5) allows for the estimation of the consumption growth rate *C** that will result in a wealth of 1000 units (the initial investment) at time *T*, conditional on the time discount for consumption (β), total return on investment (R), and initial decision date (t). The first and third columns show that consumption to income ratios of 70% restrict the consumption growth rate to levels far below the return on investment, which is exactly the issue of the equity Euler condition in the puzzle.

$R = 1.1$	$R = 1.1$	$R = 1.0698$	$R = 1.0698$
$\beta = .99$	$\beta = .99$	$\beta = .99$	$\beta = .99$
$C = 1.089$	$C = 1.089$	$C = 1.059$	$C = 1.059$
$w_t = 1000$	$w_t = 1000$	$w_t = 1000$	$w_t = 1000$
$I = 0$ (self-financing)	$I = 0$ (self-financing)	$I = 0$ (self-financing)	$I = 0$ (self-financing)
$T = 70$	$T = 70$	$T = 70$	$T = 70$
$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)	$t = 20$ (receive w_t)
$c_t(T) = 75.00$	$c_t(T) = 45$	$c_t(T) = 55$	$c_t(T) = 36$
Savings ratio = 25.00%	Savings ratio = 55.00%	Savings ratio = 21.20%	Savings ratio = 48.42%
$C^* \approx 1.018$	$C^* \approx 1.056$	$C^* \approx 1.013$	$C^* \approx 1.039$

Notes: The assumptions are identical to Table 5, except that life expectancy is 70 instead of 80.