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The Great Moderation Flattens Fat Tails: Disappearing Leptokurtosis

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Abstract

Recently, Fagiolo et al. (2008) find fat tails of economic growth rates after adjusting outliers, autocorrelation and heteroskedasticity. This paper employs US quarterly real output growth, showing that this finding of fat tails may reflect the Great Moderation. That is, leptokurtosis disappears after GARCH adjustment once we incorporate the break in the variance equation.

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1. Introduction

In a recent study, assuming that the underlying generating mechanism governing output growth dynamics exhibits time invariance, Fagiolo *et al.* (2008) find that fat tails characterize the output growth rate distribution after adjusting for outliers, autocorrelation, and heteroskedasticity for the US and other OECD countries. Using quarterly real US GDP growth, we show that once we incorporate a structural break in the variance equation for the Great Moderation, fat tails disappear in a simple autoregressive generalized autoregressive conditional heteroskedasticity (AR-GARCH) model, either under symmetric or asymmetric specifications.

We specify the AR-GARCH(1,1) model as follows:

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t, \text{ and } \varepsilon_t = \sigma_t \eta_t, \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{2}$$

where $\eta_t \sim$ iid(0,1). Bollerslev (1986) introduced this frequently employed specification that models excess kurtosis and volatility clustering in the output growth rate y_t.

The specification contained in equations (1) and (2) assumes that positive and negative shocks generate the same effect on volatility, implying a symmetric GARCH. The volatility may respond differently, however, to shocks during periods of rising or falling output growth. To provide a systematic analysis, we also examine the exponential GARCH (EGARCH) introduced by Nelson (1991) as follows:

$$log\ \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1\ log\sigma_{t-1}^2, \tag{3}$$

where asymmetry exists, if $\alpha_2 \neq 0$. When $\alpha_2 < 0$, negative shocks generate higher volatility than positive shocks of the same magnitude, and vice versa, when $\alpha_2 > 0$. The log transformation guarantees a positive variance.

When modeling real GDP growth in longer sample periods, structural changes in the mean and variance in addition to outliers will occur with a higher probability. Although no evidence of changes in the mean growth exists in studies, McConnell and Perez-Quiros (2000) among others, document a structural change in, and a rather dramatic reduction of, the volatility of US GDP growth, called the Great Moderation. The Great Moderation violates the assumption of time invariance imposed by Fagiolo *et al.* (2008).

In this study, following Franses and Ghijsels (1999), we first apply the method of Chen and Liu (1993) to detect and correct for additive outliers in GARCH models. We then use the multiple structural change test of Bai and Perron (1998, 2003a) to identify breaks in the mean and variance of the growth rate, if any. Finally, we incorporate the changes in the GARCH models to observe the behavior of leptokurtosis. In the application, we provide new evidence that leptokurtosis disappears when the break enters into the variance equation. That is, the finding of fat tails of the growth rate does not prove robust to the structural change in the variance, or the Great Moderation, which researchers, now, commonly recognize.

2. Data and Empirical Results

Output growth rates (y_t) equal the percentage change in the logarithm of seasonally adjusted quarterly real GDP, which equals nominal GDP deflated by the GDP deflator with base year 2000. All data come from the IMF *International Financial Statistics* (*IFS*) over the period 1957:1 to 2008:1.

Applying the method of Chen and Liu (1993) and the procedures developed by Franses and Ghijsels (1999), we find an additive outlier at 1978:2, which is the maximum observation in the original growth rate series. Table 1 reports summary statistics for the outlier-corrected growth rate. In Panel A, the skewness statistic displays an asymmetric distribution. The kurtosis statistic exhibits leptokurticity with fat tails. The Jarque-Bera test rejects normality. The Ljung-Box Q-statistics for the growth rates (Q) and its squared rates (Q^2) indicate autocorrelation and heteroskedasticity up to six lags, suggesting ARMA processes for the mean and the variance equations to capture the dynamic structure and to generate white-noise residuals.

We construct an AR model for the mean growth rate. The SBC selects AR(2) and this process proves adequate to produce uncorrelated residuals. Skewness, kurtosis, and heteroskedasticity, however, remain in the residuals. We adopt a GARCH-type process to capture the time-varying variance. We estimate the GARCH models employing Bollerslev and Wooldridge's (1992) quasi-maximum likelihood estimation technique, assuming normally distributed errors and using the BHHH algorithm.

In Panels B and C, the fitted GARCH and EGARCH models adequately capture the time-series properties of the growth rate. That is, the Ljung-Box Q-statistics for standardized residuals and standardized squared residuals, up to 6 lags, do not detect autocorrelation and heteroskedasticity. The residuals, however, exhibit significant leptokurtosis for the two models at the 5- and 10-percent levels. The likelihood ratio (LR) tests for $\alpha_1 + \beta_1 = 1$ in the GARCH and β_1 = 1 in the EGARCH process provide another cautionary note in that they do not reject the null hypothesis of an integrated GARCH (IGARCH) effect. The high persistence measures may reflect structural changes in the mean growth rate, its variance, or both, which the GARCH estimations ignore as shown recently by Hillebrand (2005) and Krämer and Azamo (2007).

In sum, the fat tails of the growth rate exist after we adjust for outliers, autocorrelation, and heteroskedasticity in symmetric or asymmetric GARCH models. But, so far, we ignore the Great Moderation.

Implementing the test of Bai and Perron (1998, 2003a), we find no change in the mean. In Table 2, Panel A displays the results of testing for breaks in the variance, identifying one, and only one, break in 1984:1.[1] This break date matches that identified by McConnell and Perez-Quiros (2000). In Panel B, we conduct a structural stability test, a variance-ratio test for the equality of the unconditional variances between the two sub-periods. A clear, significant decline in the standard deviation of the growth rate occurs from 1.1040 in the pre-1984 sample to 0.4934 in the post-1984 sample. The decline equals 55-percent. As the introduction notes, economists call the substantial drop in the variance of output growth in the period after the break the Great Moderation.

To examine the effect of the Great Moderation on leptokurtosis, we include a dummy variable in the conditional variance equation, which equals unity from the break date forward, zero otherwise, in the GARCH and EGARCH processes, respectively, as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma D, \tag{4}$$

$$log\ \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1\ log \sigma_{t-1}^2 + \gamma D, \tag{5}$$

where D = 1 for $t > 1984:1$; and 0, otherwise. Since the volatility declines, we expect a significant negative estimate of γ to capture the break in the variance process.

Table 3 reports the estimates with the variance dummy variable. The coefficient of the structural dummy (i.e., γ) proves significantly negative in the variance equation at the 5-percent level. The Ljung-Box Q-statistics show no evidence of autocorrelation and heteroskedasticity.

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[1] According to Bai and Perron (2003b), the $\sup F(5|0)$ test proves significant at the 5-percent level for *m=5*, suggesting the existence of at least one break in the variance. The two double maximum statistics, UD_{max} and WD_{max}, agree with this result. The test, $\sup F(2|1) = 1.6396$ falls below the critical value, suggesting that a single structural break exists at 1984:1 with a 95-percent confidence interval [1982:4-1989:3].

The coefficients of skewness and kurtosis prove insignificant. Thus, the residuals conform to a normal distribution. For both the symmetric and the asymmetric GARCH models, these results suggest that the statistical evidence for leptokurtosis in the growth rate may reflect structural change in the variance caused by the Great Moderation. Finally, the significant LR statistics indicate no IGARCH effect. That is, high volatility persistence also reflects the Great Moderation.

3. Conclusion

Using GARCH modeling, we show that fat tails of US quarterly real GDP growth rates exist after the adjustments for outliers, autocorrelation, and heteroskedasticity under the assumption of a time invariant volatility. Instability or the Great Moderation, however, governs the variance process. Once we incorporate the break into the variance equation, fat tails in the GARCH residuals disappear. This completes the unfinished tale of leptokurtosis of the output growth rate as told by Fagiolo *et al.* (2008), at least for the US.

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Table 1: Summary Statistics for Quarterly Real GDP Growth, 1957-2008

Panel A: Descriptive Statistics						
Mean	**Standard deviation**	**Maximum**	**Minimum**	**Skewness**	**Kurtosis**	**Normality test**
0.7831	0.8703	2.7320	-2.7528	-0.5644* [0.0010]	1.4517* [0.0000]	28.7443* [0.0000]
***Q* (1)**	***Q* (2)**	***Q* (3)**	***Q* (4)**	***Q* (5)**	***Q* (6)**	
19.8267* [0.0000]	27.3615* [0.0000]	27.7672* [0.0000]	27.7695* [0.0000]	32.4020* [0.0000]	32.9619* [0.0000]	
Q^2 **(1)**	Q^2 **(2)**	Q^2 **(3)**	Q^2 **(4)**	Q^2 **(5)**	Q^2 **(6)**	
8.8586* [0.0029]	16.9030* [0.0002]	20.6155* [0.0001]	21.9712* [0.0002]	22.3580* [0.0004]	22.3802* [0.0010]	
Panel B: GARCH(1,1) Estimates						
a_0	a_1	a_2				
0.4511* (0.0846)	0.2581* (0.0786)	0.1930* (0.0761)				
α_0	α_1	β_1				
0.0301* (0.0148)	0.1728* (0.0820)	0.7796* (0.0922)				
Q **(6)**	Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**	
5.9911 [0.4241]	4.8385 [0.5646]	1.8438 [0.1761]	-0.4285* [0.0135]	1.2492* [0.0003]	19.3176* [0.0000]	
Panel C: EGARCH(1,1) Estimates						
a_0	a_1	a_2				
0.4684* (0.0955)	0.1946* (0.0792)	0.2202* (0.0740)				
α_0	α_1	α_2	β_1			
-0.2908* (0.0966)	0.2934* (0.0589)	-0.0938 (0.0589)	0.9305* (0.0494)			
Q **(6)**	Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**	
4.8058 [0.5689]	7.8057 [0.2526]	1.9743 [0.1616]	-0.2125 [0.2255]	0.6329** [0.0740]	4.7967** [0.0908]	

Note: We report p-values in brackets; 0.0000 indicates less than 0.00005. The measures of skewness and kurtosis are normally distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T equals the number of observations. $Q(k)$ and $Q^2(k)$ equal Ljung-Box Q-statistics, testing for level (standardized residuals) and squared terms (squared standardized residuals) for autocorrelations up to *k* lags. Standard errors appear in parentheses. The likelihood ratio statistic (LR) tests for $\alpha_1 + \beta_1 = 1$ in GARCH and $\beta_1 = 1$ in EGARCH, respectively.

* significant at the 5-percent level.

** significant at the 10-percent level.

Table 2: Break Date and Structural Stability Test

Panel A. Structural Break Test in Variance

$Sup\ F(1\|0)$	$Sup\ F(2\|0)$	$Sup\ F(3\|0)$	$Sup\ F(4\|0)$	$Sup\ F(5\|0)$	UD_{max}	WD_{max}
44.8692*	23.0062*	15.6404*	12.3835*	11.1895*	44.8692*	44.8692*
$Sup\ F(2\|1)$	$Sup\ F(3\|2)$	$Sup\ F(4\|3)$	$Sup\ F(5\|4)$	Break date	95% Confidence Interval	
1.6396	1.0154	0.7364	0.7119	1984:1	[1982:4-1989:3]	

Panel B. Structural Stability Test

Break date	**Period**	**Mean**	**Standard Deviation**	**Sub-sample 1 vs Sub-sample 2**
1984:1	1957:1-1984:1	0.8130	1.1040	5.0059*
	1984:2-2008:1	0.7494	0.4934	[0.0000]

Note: Critical values for the structural tests are reported in Bai and Perron (2003b). The unconditional variance ratio tests for equality between the sub-samples 1 and *2*, and is asymptotically distributed as $F(df_1, df_2)$, where *df* denotes the degrees of freedom. The p-value 0.0000 in the bracket indicates less than 0.00005.

* significant at the 5-percent level.

Table 3: GARCH(1,1) Estimates with Structural Break in Variance

Panel A: GARCH(1,1) Estimates

a_0	a_1	a_2			
0.4638*	0.2139*	0.1770*			
(0.0793)	(0.0714)	(0.0715)			
α_0	α_1	β_1	γ		
0.9306*	0.0981	0.1004	-0.7754*		
(0.3287)	(0.0899)	(0.2656)	(0.2801)		
Q **(6)**	Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
2.5908	5.1249	9.2517*	-0.1629	-0.1508	1.0850
[0.8581]	[0.5278]	[0.0027]	[0.3481]	[0.6670]	[0.5812]

Panel B: EGARCH(1,1) Estimates

a_0	a_1	a_2			
0.4442*	0.2665*	0.1605*			
(0.0872)	(0.0788)	(0.0713)			
α_0	α_1	α_2	β_1	γ	
0.0752	-0.0072	0.0848	-0.5500	-2.5057*	
(0.2217)	(0.1420)	(0.0886)	(0.3714)	(0.5365)	
Q **(6)**	Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
2.4032	7.9690	17.4114*	-0.2645	0.1013	2.4419
[0.8791]	[0.2403]	[0.0000]	[0.1276]	[0.7725]	[0.2949]

Note: See Table 1.

* significant at the 5-percent level.